Closing *Thursday*: 2.5-6
Closing Friday: 2.7
Please visit office hours
1:30-3:00pm in PDL C-339

When we write

$$\lim_{x\to\infty} f(x) = L$$

we say "the limit of f(x), as x goes to infinity is L", and we mean

as *x* takes on larger and larger positive numbers,
y = f(x) takes on values closer and closer to L.

2.6 Limits "at" Infinity (*Horizontal Asymptotes*)

Goal: Study "long term" behavior.

Entry Task: What are these limits?

For any <u>positive</u> number *n*,

1. $\lim_{x\to\infty} x^{-n} = \lim_{x\to\infty} \frac{1}{x^n} =$

$\lim_{x\to-\infty} x^{-n} = \lim_{x\to-\infty} \frac{1}{x^n} =$
(if defined)

2. $\lim_{x\to\infty} e^{x} =$ and $\lim_{x\to\infty} e^{-x} =$

$\lim_{x\to-\infty} e^{x} =$ and $\lim_{x\to-\infty} e^{-x} =$

3. $\lim_{x\to\infty} \ln(x) =$

4. $\lim_{x\to\infty} \tan^{-1}(x) =$,

$\lim_{x\to-\infty} \tan^{-1}(x) =$

Strategies to compute

$$\lim_{x\to\infty} f(x)$$

1. Is it a limit from entry task?

   If so, *done*. If not, go to next step.

2. Rewrite in terms of known limits:

Strategy 1: Multiply top/bot by $\frac{1}{x^a}$, where a is the largest power.

Strategy 2: Multiply top/bot by $\frac{1}{e^{rx}}$.

Strategy 3: Multiply by conjugate.

Strategy 4: Combine into one fraction.

Strategy 5: Change variable

Note about roots

$$\sqrt{x^2} = \quad x, \text{if } x \geq 0, \text{ and}$$
$$\sqrt{x^2} = -x, \text{if } x < 0.$$

1. $\displaystyle \lim_{x\to\infty} \frac{3+5e^{(2x)}}{2e^x+4e^{(2x)}}$

2. $\displaystyle \lim_{x\to\infty} \frac{\sqrt{9x^6-x+1}}{2x^3-x^2}$

3. $\displaystyle \lim_{x\to-\infty} \frac{\sqrt{9x^6-x+1}}{2x^3-x^2}$

4. $\lim_{x \to \infty} \left(\sqrt{3 + 2x + x^2} - x\right)$

2.7 Introduction to Derivative